Analysis of the triply heavy baryon states with QCD sum rules

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Abstract

In this article, we study the $\frac{1}{2}^{\pm}$ and $\frac{3}{2}^{\pm}$ triply heavy baryon states in a systematic way by subtracting the contributions from the corresponding $\frac{1}{2}^{\mp}$ and $\frac{3}{2}^{\mp}$ triply heavy baryon states with the QCD sum rules, and make reasonable predictions for their masses.

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Key words: Triply heavy baryon states, QCD sum rules

1 Introduction

By this time, the $\frac{1}{2}^{+}$ and $\frac{1}{2}^{-}$ antitriplet charmed baryon states (Λ_c^+, Ξ_c^+, Ξ_c^0) and ($\Lambda_c^+(2595)$, $\Xi_c^+(2790)$, $\Xi_c^0(2790)$), and the $\frac{1}{2}^{+}$ and $\frac{3}{2}^{+}$ sextet charmed baryon states ($\Omega_c, \Sigma_c, \Xi_c'$) and ($\Omega_c^*, \Sigma_c^*, \Xi_c^*$) have been observed, while the S-wave bottom baryon states are far from complete, only the Λ_b^0, Σ_b^+, Σ_b^0, Σ_b^-, Σ_b^{*+}, Σ_b^{*-}, Ξ_b^0, Ξ_b^- and Ω_b^- have been observed [1]. In 2002, the SELEX collaboration reported the first observation of the doubly charmed baryon state Ξ_{cc}^+ in the decay $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ [2], and confirmed it later in the decay $\Xi_{cc}^+ \to pD^+K^-$ [3]. However, the Babar collaboration observed no evidence for the Ξ_{cc}^+ in the $\Lambda_c^+K^-\pi^+$, $\Xi_c^0\pi^+$ decay modes and for the Ξ_{cc}^{++} in the $\Lambda_c^+K^-\pi^+\pi^+$, $\Xi_c^0\pi^+\pi^+$ decay modes, and the Belle collaboration observed no evidence for the Ξ_{cc}^+ in the $\Lambda_c^+K^-\pi^+$ decay mode [4, 5]. There are no experimental signals for the triply heavy baryon states, we expect that the large hadron collider (LHC) will provide us with the whole heavy, doubly heavy and triply heavy baryon states [6, 7].

The triply heavy baryon states and heavy quarkonium states play an important role in understanding the heavy quark dynamics at the hadronic scale due to the absence of the light quark contaminations, and serve as an excellent subject in studying the interplay between the perturbative and nonperturbative QCD. On the other hand, the QCD sum rules is a powerful nonperturbative theoretical tool in studying the ground state hadrons [8, 9]. In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties. Taking the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [8, 9]. There have been many works on the masses of the heavy and doubly heavy baryon states with the QCD sum rules [10, 11]. It is interesting to study the mass spectrum of the triply heavy baryon states using the QCD sum rules.

In Ref.[11], we take the novel approach introduced by Jido et al [12] to study the positive-parity and negative-parity heavy and doubly heavy baryons in a systematic way by separating the contributions of the positive-parity and negative-parity baryons explicitly, as the interpolating currents have non-vanishing couplings to both the positive-parity and

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negative-parity baryons, there exist contaminations [13]. Before the work of Jido et al, Bagan et al take the heavy quark limit to separate the contributions of the positive-parity and negative-parity heavy baryons unambiguously [14]. In this article, we extend our previous works to study the $\frac{1}{2}^{\pm}$ and $\frac{3}{2}^{\pm}$ triply heavy baryon states by subtracting the contributions from the corresponding $\frac{1}{2}^{\mp}$ and $\frac{3}{2}^{\mp}$ triply heavy baryon states with the full QCD sum rules.

The existing theoretical works focus on the heavy and doubly heavy baryon states, the works on the triply heavy baryon states are relatively few, for example, the effective field theory [15], the lattice QCD [16], the QCD bag model [17], the quark model estimation [18], the variational approach [19], the modified bag model [20], the relativistic three-quark model [21], the QCD sum rules [22], the non-relativistic three-quark model [23, 24, 25], potential non-relativistic QCD [26], the Regge trajectory ansatz [27], etc.

The article is arranged as follows: we derive the QCD sum rules for the masses and the pole residues of the triply heavy baryon states in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

2 QCD sum rules for the triply heavy baryon states

The $\frac{1}{2}^+$ and $\frac{3}{2}^+$ triply heavy baryon states $\Omega_{QQQ'}(\frac{1}{2})$, $\Omega_{QQQ'}(\frac{3}{2})$ and $\Omega_{QQQ}(\frac{3}{2})$ can be interpolated by the triply heavy quark currents $J^{QQQ'}(x)$, $J_\mu^{QQQ'}(x)$ and $J_\mu^{QQQ}(x)$, respectively,

$$\begin{aligned}
J^{QQQ'}(x) &= \epsilon^{ijk} Q_i^T(x) C\gamma_\mu Q_j(x) \gamma^\mu \gamma_5 Q'_k(x) \, , \\
J_\mu^{QQQ'}(x) &= \epsilon^{ijk} Q_i^T(x) C\gamma_\mu Q_j(x) Q'_k(x) \, , \\
J_\mu^{QQQ}(x) &= \epsilon^{ijk} Q_i^T(x) C\gamma_\mu Q_j(x) Q_k(x) \, ,
\end{aligned} \tag{1}$$

where the Q and Q' represent the heavy quarks c and b, the i, i and k are color indexes, and the C is the charge conjunction matrix. There are other currents such as the $\eta^{QQQ'}$ besides the Ioffe currents $J^{QQQ'}$ to interpolate the $\frac{1}{2}^+$ triply heavy baryon states,

$$\eta^{QQQ'}(x) = \epsilon^{ijk} Q_i^T(x) C\sigma_{\mu\nu} Q_j(x) \sigma^{\mu\nu} \gamma_5 Q'_k(x) \, . \tag{2}$$

The currents $J^{QQQ'}$ and $\eta^{QQQ'}$ correspond to the superimpositions $\mathcal{O}_1^{QQQ'} - \mathcal{O}_2^{QQQ'}$ and $\mathcal{O}_1^{QQQ'} + \mathcal{O}_2^{QQQ'}$ respectively, where the fundamental currents $\mathcal{O}_1^{QQQ'}$ and $\mathcal{O}_2^{QQQ'}$ are defined as

$$\begin{aligned}
\mathcal{O}_1^{QQQ'}(x) &= \epsilon^{ijk} Q_i^T(x) C Q'_j(x) \gamma_5 Q_k(x) \, , \\
\mathcal{O}_2^{QQQ'}(x) &= \epsilon^{ijk} Q_i^T(x) C\gamma_5 Q'_j(x) Q_k(x) \, .
\end{aligned} \tag{3}$$

We can take the simple replacements $Q \to u$ and $Q' \to d$ to obtain the corresponding currents for the proton [13, 28]. The convergent behavior of the current η^{uud} is not as good as the Ioffe current J^{uud}, and appearance of chirality violation terms in the operator product expansion indicates that the current η^{uud} couples strongly both to the positive-parity and negative-parity baryon states [29]. We can also choose the most general current $\mathcal{O}^{uud}$,

$$\mathcal{O}^{uud}(x) = \mathcal{O}_1^{uud}(x) + t\mathcal{O}_2^{uud}(x) \, , \tag{4}$$

and search for the ideal value t. Detailed studies of all the octet baryon states based on the QCD sum rules indicate that the optimal value is about $t=-1$, if the experimental values of the masses are taken as the input parameters [30], i.e. the Ioffe currents work well. We expect that the conclusion survives in the case of the heavy quark systems. At the present time, no experimental data are available to be taken as the input parameters in searching for the optimal value of the t.

The corresponding $\frac{1}{2}^-$ and $\frac{3}{2}^-$ triply heavy baryon states can be interpolated by the currents $J^-=i\gamma_5 J^+$ and $J^-_\mu=i\gamma_5 J^+_\mu$ because multiplying $i\gamma_5$ to the currents J^+ and J^+_μ changes their parity [12], where the currents J^+ and J^+_μ denotes the triply heavy quark currents $J^{QQQ'}(x)$, $J^{QQQ'}_\mu(x)$ and $J^{QQQ}_\mu(x)$, respectively.

The correlation functions $\Pi^\pm(p)$ and $\Pi^\pm_{\mu\nu}(p)$ are defined by

$$\begin{aligned}\Pi^{\pm}(p) &= i\int d^4x e^{ip\cdot x}\langle 0|T\left\{J^{\pm}(x)\bar{J}^{\pm}(0)\right\}|0\rangle\,,\\ \Pi^{\pm}_{\mu\nu}(p) &= i\int d^4x e^{ip\cdot x}\langle 0|T\left\{J^{\pm}_\mu(x)\bar{J}^{\pm}_\nu(0)\right\}|0\rangle\,.\end{aligned}\tag{5}$$

The currents $J^\pm(x)$ couple to the $\frac{1}{2}^\pm$ triply heavy baryon states $B_\pm$, while the currents $J^\pm_\mu(x)$ couple to both the $\frac{3}{2}^\pm$ triply heavy baryon states $B^*_\pm$ and the $\frac{1}{2}^\pm$ triply heavy baryon states $B_\pm$ [13],

$$\begin{aligned}\langle 0|J^+(0)|B_\pm(p)\rangle\langle B_\pm(p)|\bar{J}^+(0)|0\rangle &= -\gamma_5\langle 0|J^-(0)|B_\pm(p)\rangle\langle B_\pm(p)|\bar{J}^-(0)|0\rangle\gamma_5\,,\\ \langle 0|J^+_\mu(0)|B^*_\pm(p)\rangle\langle B^*_\pm(p)|\bar{J}^+_\nu(0)|0\rangle &= -\gamma_5\langle 0|J^-_\mu(0)|B^*_\pm(p)\rangle\langle B^*_\pm(p)|\bar{J}^-_\nu(0)|0\rangle\gamma_5\,,\\ \langle 0|J^+_\mu(0)|B_\pm(p)\rangle\langle B_\pm(p)|\bar{J}^+_\nu(0)|0\rangle &= -\gamma_5\langle 0|J^-_\mu(0)|B_\pm(p)\rangle\langle B_\pm(p)|\bar{J}^-_\nu(0)|0\rangle\gamma_5\,,\end{aligned}\tag{6}$$

where

$$\begin{aligned}\langle 0|J^{\pm}(0)|B_\pm(p)\rangle &= \lambda_\pm U(p,s)\,,\\ \langle 0|J^{\pm}_\mu(0)|B^*_\pm(p)\rangle &= \lambda_\pm U_\mu(p,s)\,,\\ \langle 0|J^{\pm}_\mu(0)|B_\mp(p)\rangle &= \lambda_\mp\left(\gamma_\mu-4\frac{p_\mu}{M_\mp}\right)U(p,s)\,,\end{aligned}\tag{7}$$

the $\lambda_\pm$ are the pole residues, the $M_\pm$ are the masses, and the Dirac spinors $U(p,s)$ and $U_\mu(p,s)$ satisfy the following identifies,

$$\begin{aligned}\sum_s U(p,s)\overline{U}(p,s) &= \not{p}+M_\pm\,,\\ \sum_s U_\mu(p,s)\overline{U}_\nu(p,s) &= (\not{p}+M_\pm)\left[-g_{\mu\nu}+\frac{\gamma_\mu\gamma_\nu}{3}+\frac{2p_\mu p_\nu}{3M^2_\pm}-\frac{p_\mu\gamma_\nu-p_\nu\gamma_\mu}{3M_\pm}\right]\,.\end{aligned}\tag{8}$$

We insert a complete set of intermediate triply heavy baryon states with the same quantum numbers as the current operators $J^\pm(x)$ and $J^\pm_\mu(x)$ into the correlation functions $\Pi^\pm(p)$ and $\Pi^\pm_{\mu\nu}(p)$ to obtain the hadronic representation [8]. After isolating the pole terms of the lowest states of the positive-parity and negative-parity triply heavy baryons, we

obtain the following results [12]:

$$
\begin{aligned}
\Pi^{\pm}(p) &= \lambda_{+}^{2}\frac{\not\!p+M_{+}}{M_{+}^{2}-p^{2}}+\lambda_{-}^{2}\frac{\not\!p-M_{-}}{M_{-}^{2}-p^{2}}+\cdots\,,\\
\Pi^{\pm}_{\mu\nu}(p) &= -\Pi_{\pm}(p)g_{\mu\nu}+\cdots\,,\\
\Pi_{\pm}(p) &= \lambda_{+}^{2}\frac{\not\!p+M_{+}}{M_{+}^{2}-p^{2}}+\lambda_{-}^{2}\frac{\not\!p-M_{-}}{M_{-}^{2}-p^{2}}+\cdots\,. \qquad (9)
\end{aligned}
$$

In this article, we choose the tensor structure $g_{\mu\nu}$ for analysis, the $\frac{1}{2}^{\pm}$ triply heavy baryon states have no contaminations.

We can take $\vec{p}=0$ for the correlation functions $\Pi(p)$ ($\Pi^{\pm}(p)$, $\Pi_{\pm}(p)$), and obtain the spectral densities at the phenomenological side,

$$
\begin{aligned}
\mathrm{limit}_{\epsilon\rightarrow0}\frac{\mathrm{Im}\Pi(\mathrm{p_0+i\epsilon})}{\pi} &= \lambda_{+}^{2}\frac{\gamma_0+1}{2}\delta(p_0-M_{+})+\lambda_{-}^{2}\frac{\gamma_0-1}{2}\delta(p_0-M_{-})+\cdots\\
&= \gamma_0A(p_0)+B(p_0)+\cdots\,, \qquad (10)
\end{aligned}
$$

where

$$
\begin{aligned}
A(p_0) &= \frac{1}{2}\left[\lambda_{+}^{2}\delta(p_0-M_{+})+\lambda_{-}^{2}\delta(p_0-M_{-})\right]\,,\\
B(p_0) &= \frac{1}{2}\left[\lambda_{+}^{2}\delta(p_0-M_{+})-\lambda_{-}^{2}\delta(p_0-M_{-})\right]\,, \qquad (11)
\end{aligned}
$$

the combinations $A(p_0)+B(p_0)$ and $A(p_0)-B(p_0)$ contain the contributions from the positive-parity and negative-parity triply heavy baryon states, respectively.

In the following, we briefly outline the operator product expansion performed at the large space-like region $p^2 \ll 0$. We contract the heavy quark fields in the correlation functions $\Pi^{\pm}(p)$ and $\Pi^{\pm}_{\mu\nu}(p)$ with Wick theorem, substitute the full heavy quark propagators into the correlation functions $\Pi^{\pm}(p)$ and $\Pi^{\pm}_{\mu\nu}(p)$ firstly, then complete the integrals in the coordinate space and momentum space sequentially to obtain the correlation functions $\Pi^{\pm}(p)$ and $\Pi^{\pm}_{\mu\nu}(p)$ at the quark level. In calculations, we take into account all diagrams like the typical ones shown in Fig.1. Once the analytical quark-level correlation functions $\Pi^{\pm}(p)$ and $\Pi^{\pm}_{\mu\nu}(p)$ are obtained, we take the limit $\vec{p}=0$, and use the dispersion relation to obtain the QCD spectral densities $\rho^A(p_0)$ and $\rho^B(p_0)$ (which correspond to the tensor structures γ_0 and 1 respectively). Finally we introduce the weight functions $\exp\left[-\frac{p_0^2}{T^2}\right]$, $p_0^2\exp\left[-\frac{p_0^2}{T^2}\right]$, and obtain the following QCD sum rules,

$$
\lambda^2_{\pm}\exp\left[-\frac{M_{\pm}^2}{T^2}\right] = \int_{\Delta}^{\sqrt{s_0}}dp_0\left[\rho^A(p_0)\pm\rho^B(p_0)\right]\exp\left[-\frac{p_0^2}{T^2}\right]\,, \qquad (12)
$$

$$
\lambda^2_{\pm}M_{\pm}^2\exp\left[-\frac{M_{\pm}^2}{T^2}\right] = \int_{\Delta}^{\sqrt{s_0}}dp_0\left[\rho^A(p_0)\pm\rho^B(p_0)\right]p_0^2\exp\left[-\frac{p_0^2}{T^2}\right]\,, \qquad (13)
$$

where the s_0 are the continuum threshold parameters, the T^2 are the Borel parameters, and the threshold parameters $\Delta=2m_Q+m_{Q'}$ or $3m_Q$. The spectral densities $\rho^A(p_0)$ and $\rho^B(p_0)$ at the level of quark-gluon degrees of freedom are given explicitly in the Appendix. We can obtain the masses $M_{\pm}$ and pole residues $\lambda_{\pm}$ by solving above equations with simultaneous iterations.

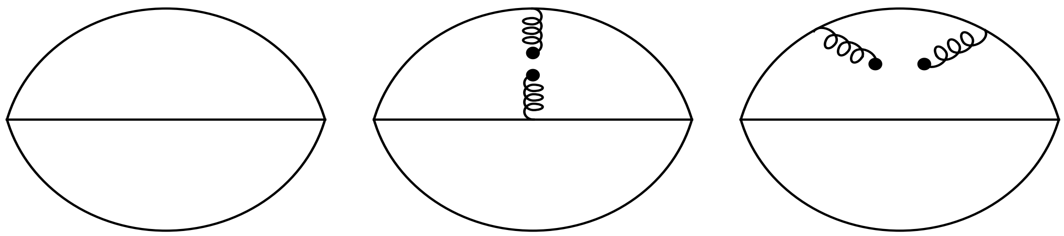

Figure 1: The typical diagrams we calculate in the operator product expansion, we take into account the tree-level perturbative term and the gluon condensates.

3 Numerical results and discussions

The input parameters are taken as $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.012 \pm 0.004)\,\mathrm{GeV}^4$ [31], $m_c = 1.3\,\mathrm{GeV}$ and $m_b = 4.7\,\mathrm{GeV}$ [1]. The value of the gluon condensate $\langle\frac{\alpha_s GG}{\pi}\rangle$ has been updated from time to time, and changes greatly [9]. The updated value $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.023 \pm 0.003)\,\mathrm{GeV}^4$ [9] and the standard value $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.012 \pm 0.004)\,\mathrm{GeV}^4$ [31] lead to a tiny difference as the gluon condensate makes tiny contribution. The heavy quark masses appearing in the perturbative terms are usually taken to be the pole masses in the QCD sum rules, while the choice of the m_Q in the leading-order coefficients of the higher-dimensional terms is arbitrary [9, 32]. In calculations, we observe that the dominating contributions come from the perturbative term. So we take the pole masses and neglect the uncertainties of the pole masses. The integral intervals of the energy p_0 are rather small, variations of the threshold parameters $\Delta = (2m_Q + m_{Q'})$ or $3m_Q$ can lead to remarkable changes of the continuum threshold parameters $\sqrt{s_0}$, we can fix the Δ and vary the $\sqrt{s_0}$.

In the conventional QCD sum rules [8], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter T^2 and continuum threshold parameter s_0. We impose the two criteria on the triply heavy baryon states to choose the Borel parameter T^2 and continuum threshold parameter s_0. In our previous works on the heavy and doubly heavy baryon states, the pole contributions are taken as $(45-80)\%$ [11]. We can take the same pole contributions, then search for the continuum threshold parameters $\sqrt{s_0}$ to reproduce the relation $\sqrt{s_0} = M_\pm + (0.4 \sim 0.6)\,\mathrm{GeV}$. The Borel parameters T^2, continuum threshold parameters $\sqrt{s_0}$, masses, pole residues are shown in Table 1 and Figs.2-3. In this article, we have neglected the contributions of the perturbative $\mathcal{O}(\alpha_s)$ corrections, which can be taken into account by introducing formal coefficient $1 + \frac{\alpha_s}{\pi} f(m_Q, m_{Q'}, s_0)$ through the unknown function $f(m_Q, m_{Q'}, s_0)$. As the dominant contributions come from the perturbative term, we expect that the $\mathcal{O}(\alpha_s)$ corrections to the perturbative term cannot change the masses remarkably, those effects can be absorbed in the pole residues approximately.

If we choose the structures γ_0 and 1 to study the masses, there are contaminations of the negative- (or positive-) parity triply heavy baryon states to the positive- (or negative-) parity triply heavy baryon states, the corresponding fractions can be expressed as

$$R_\pm = \frac{\int_\Delta^{\sqrt{s_0}} dp_0 \left[\rho^A(p_0) \mp \rho^B(p_0)\right] \exp\left[-\frac{p_0^2}{T^2}\right]}{\int_\Delta^{\sqrt{s_0}} dp_0 \left[\rho^A(p_0) \pm \rho^B(p_0)\right] \exp\left[-\frac{p_0^2}{T^2}\right]} \,. \tag{14}$$

In this article, we separate the contributions of the positive-parity and negative-parity

triply heavy baryon states explicitly.

In calculations, we have taken the pole masses. On the other hand, we can take the $\overline{MS}$ masses $m_c(m_c^2) = 1.2\,\text{GeV}$, $m_b(m_b^2) = 4.2\,\text{GeV}$, as the $\overline{MS}$ masses are also used in the QCD sum rules, for example, in studying the $B \to \pi$ form-factors [33]. We choose the same Borel parameters and suitable continuum threshold parameters to reproduce the same pole contributions so as to obtain the ground state masses and pole residues, the predictions are presented in Table 1. From Table 1, we can see that the pole masses and the $\overline{MS}$ masses result in large discrepancies for the masses of the triply heavy baryon states. If the $\overline{MS}$ masses are taken, the present predictions are compatible with the values from Ref.[22] within uncertainties. In Ref.[22], the contributions of the positive-parity baryon states are not distinguished from that of the negative-parity baryon states. For the established bottom baryon states Σ_b with three stars, the masses are $M_{\Sigma_b^+} = 5.8078\,\text{GeV}$ and $M_{\Sigma_b^-} = 5.8152\,\text{GeV}$ respectively from the Particle Data Group [1]. The prediction $M_{\Sigma_b} = (5.80 \pm 0.19)\,\text{GeV}$ based on the pole mass $m_b = (4.7 \pm 0.1)\,\text{GeV}$ is consistent with experimental data [11], while the prediction $M_{\Sigma_b} = (5.72 \pm 0.19)\,\text{GeV}$ based on the $\overline{MS}$ mass $m_b(m_b^2) = (4.2 \pm 0.1)\,\text{GeV}$ underestimates the experimental data about $80\,\text{MeV}$, if we take the same values of other parameters; so the pole masses are preferred. In the QCD sum rules, if the variations of the threshold parameters Δ can lead to relatively large changes for the integral ranges $\sqrt{s_0} - \Delta$, the predictions are sensitive to the masses m_Q. In the present case, the mass uncertainty $\delta m_b = 0.1\,\text{GeV}$ can result in uncertainty $\frac{\delta\Delta}{\sqrt{s_0}-\Delta} \approx 20\%$ for the triply-bottom baryon state Ω_{bbb}. The pole mass and the $\overline{MS}$ mass correspond to quite different continuum threshold parameters s_0, see Table 1. On the other hand, if the variations of the threshold parameters Δ result in small values of the $\frac{\delta\Delta}{\sqrt{s_0}-\Delta}$, the predictions based on the pole masses and the $\overline{MS}$ masses lead to small discrepancies. Irrespective of the pole masses and the $\overline{MS}$ masses, it would be better to understand the heavy quark masses m_Q as the effective quark masses (or just the mass parameters). Our previous works on the mass spectrum of the $\frac{1}{2}^{\pm}$ and $\frac{3}{2}^{\pm}$ heavy and doubly heavy baryon states indicate such parameters (the pole masses) can lead to satisfactory results [11], we prefer the pole masses.

There are no experimental data for the masses of the triply heavy baryon states, the present predictions are compared with other theoretical calculations, such as the QCD bag model [17], the quark model estimation [18], the variational approach [19], the modified bag model [20], the relativistic three-quark model [21], the QCD sum rules [22], the non-relativistic three-quark model [23], see Table 2. All those predictions should be confronted with the experimental data in the future. The LHC will be the world's most copious source of the b hadrons, and a complete spectrum of the b and c hadrons will be available through the gluon fusions. In proton-proton collisions at $\sqrt{s} = 14\,\text{TeV}$, the $b\bar{b}$ cross section is expected to be $\sim 500\mu b$ producing 10^{12} $b\bar{b}$ pairs in a standard year of running at the LHCb operational luminosity of $2 \times 10^{32}\text{cm}^{-2}\text{sec}^{-1}$ [6].

4 Conclusion

In this article, we extend our previous works on the mass spectrum of the heavy and doubly heavy baryon states to study the $\frac{1}{2}^{\pm}$ and $\frac{3}{2}^{\pm}$ triply heavy baryon states by subtracting the contributions from the corresponding $\frac{1}{2}^{\mp}$ and $\frac{3}{2}^{\mp}$ triply heavy baryon states with the

	$T^2(\mathrm{GeV}^2)$	$\sqrt{s_0}(\mathrm{GeV})$	pole	$M(\mathrm{GeV})$	$\lambda(\mathrm{GeV}^3)$
$\Omega_{ccc}(\frac{3}{2}^+)$	$4.6-6.4$	5.6 ± 0.2	$(41-79)\%$	4.99 ± 0.14	0.20 ± 0.04
$\Omega_{ccb}(\frac{1}{2}^+)$	$6.3-8.3$	8.8 ± 0.2	$(43-80)\%$	8.23 ± 0.13	0.47 ± 0.10
$\Omega_{ccb}(\frac{3}{2}^+)$	$6.4-8.4$	8.8 ± 0.2	$(43-80)\%$	8.23 ± 0.13	0.26 ± 0.05
$\Omega_{bbc}(\frac{1}{2}^+)$	$8.0-10.0$	12.0 ± 0.2	$(44-79)\%$	11.50 ± 0.11	0.68 ± 0.15
$\Omega_{bbc}(\frac{3}{2}^+)$	$8.0-10.0$	12.0 ± 0.2	$(45-80)\%$	11.49 ± 0.11	0.39 ± 0.09
$\Omega_{bbb}(\frac{3}{2}^+)$	$10.0-12.0$	15.3 ± 0.2	$(45-79)\%$	14.83 ± 0.10	0.68 ± 0.16
$\Omega_{ccc}(\frac{3}{2}^-)$	$5.1-7.1$	5.8 ± 0.2	$(44-80)\%$	5.11 ± 0.15	0.24 ± 0.04
$\Omega_{ccb}(\frac{1}{2}^-)$	$7.2-9.2$	9.0 ± 0.2	$(46-79)\%$	8.36 ± 0.13	0.57 ± 0.11
$\Omega_{ccb}(\frac{3}{2}^-)$	$7.3-9.3$	9.0 ± 0.2	$(47-79)\%$	8.36 ± 0.13	0.32 ± 0.06
$\Omega_{bbc}(\frac{1}{2}^-)$	$9.5-11.5$	12.2 ± 0.2	$(46-77)\%$	11.62 ± 0.11	0.86 ± 0.17
$\Omega_{bbc}(\frac{3}{2}^-)$	$9.5-11.5$	12.2 ± 0.2	$(47-78)\%$	11.62 ± 0.11	0.49 ± 0.10
$\Omega_{bbb}(\frac{3}{2}^-)$	$11.4-14.0$	15.5 ± 0.2	$(48-80)\%$	14.95 ± 0.11	0.86 ± 0.17
$\overline{\Omega}_{ccc}(\frac{3}{2}^+)$	$4.6-6.4$	5.4 ± 0.2	$(42-79)\%$	4.76 ± 0.14	0.20 ± 0.04
$\overline{\Omega}_{ccb}(\frac{1}{2}^+)$	$6.3-8.3$	8.2 ± 0.2	$(42-78)\%$	7.61 ± 0.13	0.47 ± 0.10
$\overline{\Omega}_{ccb}(\frac{3}{2}^+)$	$6.4-8.4$	8.2 ± 0.2	$(43-79)\%$	7.60 ± 0.13	0.26 ± 0.05
$\overline{\Omega}_{bbc}(\frac{1}{2}^+)$	$8.0-10.0$	11.0 ± 0.2	$(43-78)\%$	10.47 ± 0.12	0.68 ± 0.15
$\overline{\Omega}_{bbc}(\frac{3}{2}^+)$	$8.0-10.0$	11.0 ± 0.2	$(44-78)\%$	10.46 ± 0.12	0.39 ± 0.09
$\overline{\Omega}_{bbb}(\frac{3}{2}^+)$	$10.0-12.0$	13.9 ± 0.2	$(45-78)\%$	13.40 ± 0.10	0.66 ± 0.15
$\overline{\Omega}_{ccc}(\frac{3}{2}^-)$	$5.1-7.1$	5.6 ± 0.2	$(44-80)\%$	4.88 ± 0.15	0.24 ± 0.04
$\overline{\Omega}_{ccb}(\frac{1}{2}^-)$	$7.2-9.2$	8.4 ± 0.2	$(45-78)\%$	7.74 ± 0.13	0.57 ± 0.11
$\overline{\Omega}_{ccb}(\frac{3}{2}^-)$	$7.3-9.3$	8.4 ± 0.2	$(46-78)\%$	7.73 ± 0.13	0.32 ± 0.06
$\overline{\Omega}_{bbc}(\frac{1}{2}^-)$	$9.5-11.5$	11.2 ± 0.2	$(45-75)\%$	10.60 ± 0.12	0.84 ± 0.17
$\overline{\Omega}_{bbc}(\frac{3}{2}^-)$	$9.5-11.5$	11.2 ± 0.2	$(46-76)\%$	10.59 ± 0.11	0.47 ± 0.10
$\overline{\Omega}_{bbb}(\frac{3}{2}^-)$	$11.4-14.0$	14.1 ± 0.2	$(46-78)\%$	13.52 ± 0.11	0.82 ± 0.16

Table 1: The Borel parameters, continuum threshold parameters, pole contributions, masses and pole residues of the triply heavy baryon states. The overline on the $\Omega_{QQQ'}$ denotes the $\overline{MS}$ masses are used.

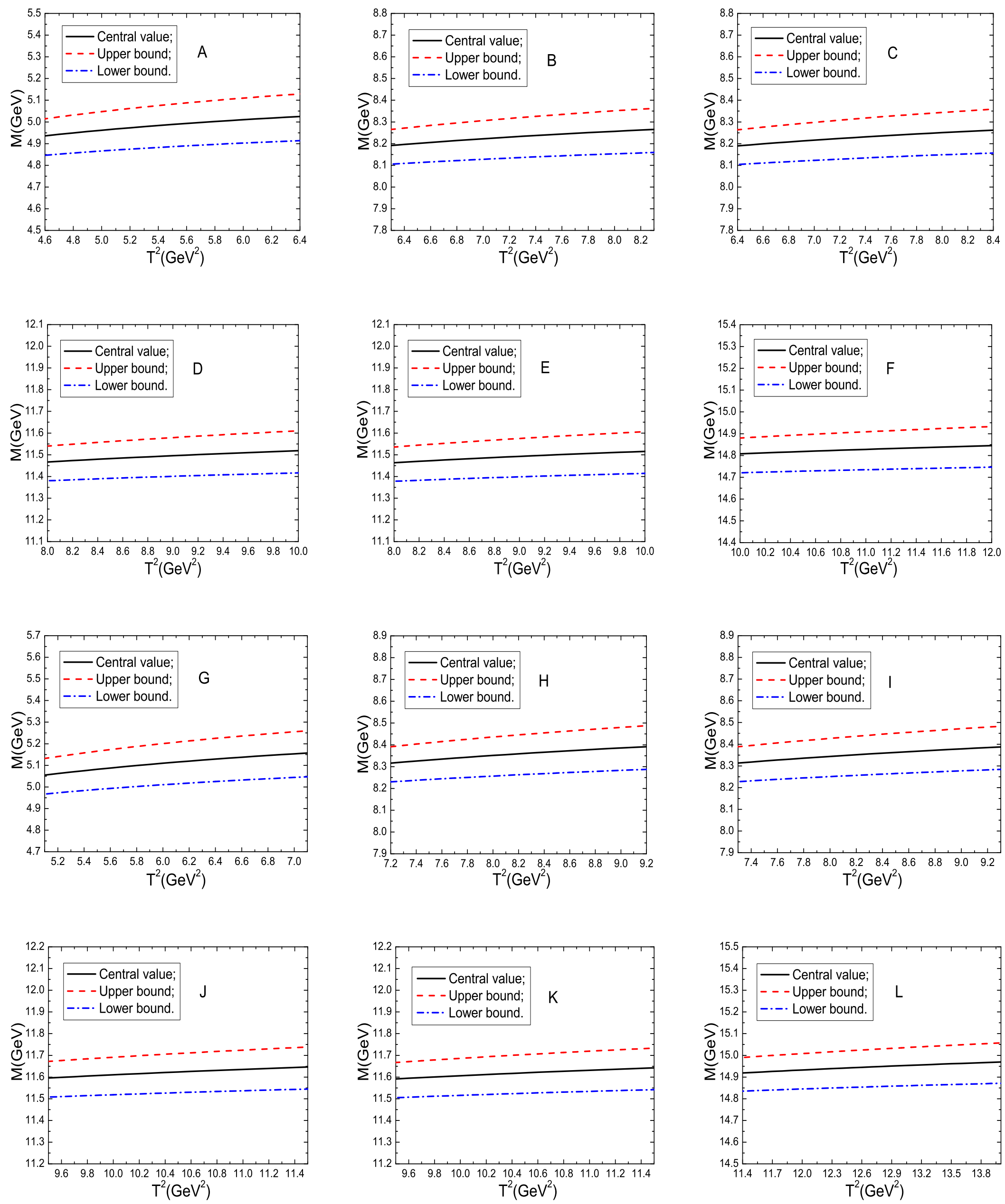

Figure 2: The masses of the triply heavy baryon states with variations of the Borel parameters, the A, B, C, D, E, F, G, H, I, J, K and L correspond to the $\Omega_{ccc}(\frac{3}{2}^+)$, $\Omega_{ccb}(\frac{1}{2}^+)$, $\Omega_{ccb}(\frac{3}{2}^+)$, $\Omega_{bbc}(\frac{1}{2}^+)$, $\Omega_{bbc}(\frac{3}{2}^+)$, $\Omega_{bbb}(\frac{3}{2}^+)$, $\Omega_{ccc}(\frac{3}{2}^-)$, $\Omega_{ccb}(\frac{1}{2}^-)$, $\Omega_{ccb}(\frac{3}{2}^-)$, $\Omega_{bbc}(\frac{1}{2}^-)$, $\Omega_{bbc}(\frac{3}{2}^-)$ and $\Omega_{bbb}(\frac{3}{2}^-)$, respectively.

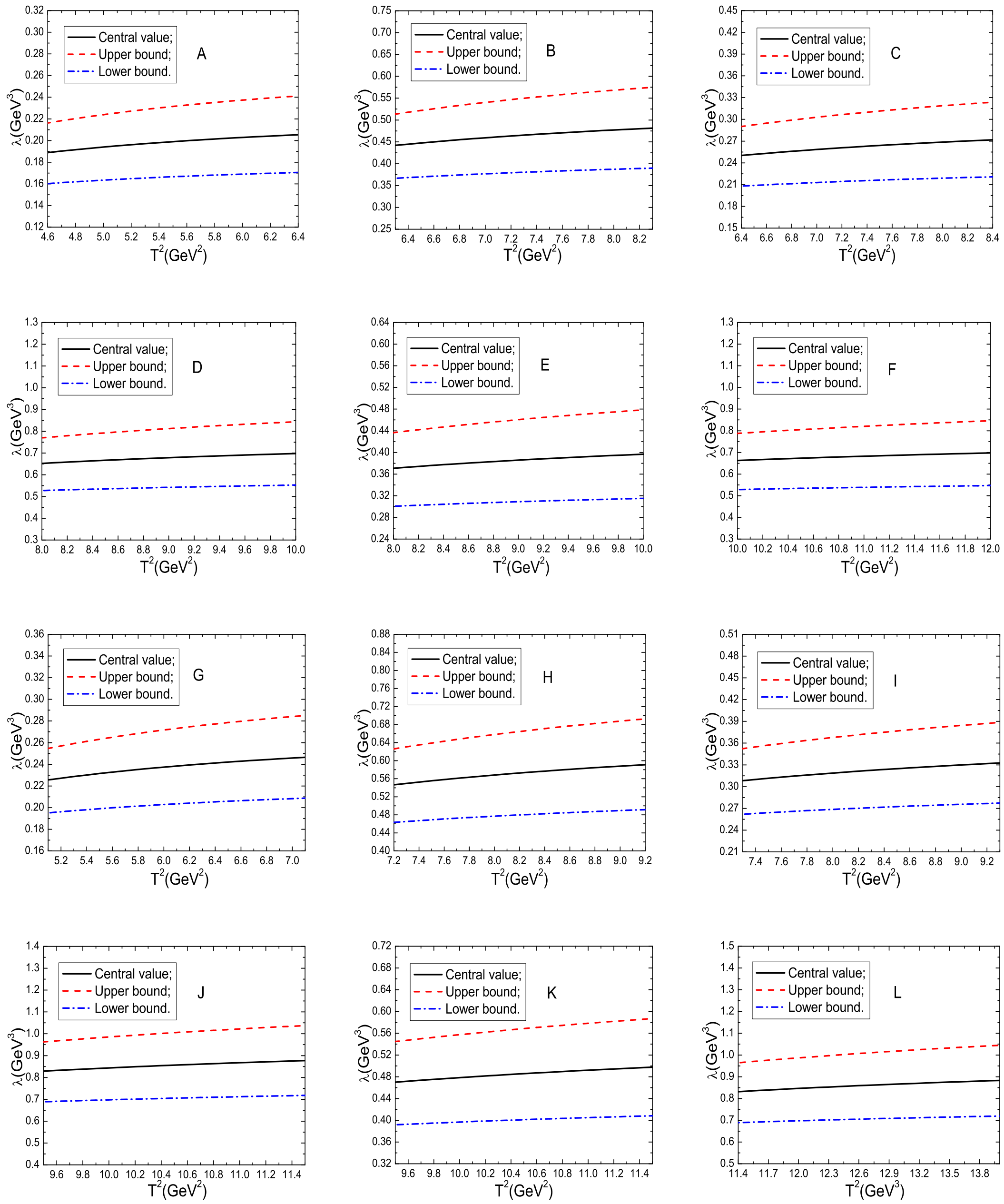

Figure 3: The pole residues of the triply heavy baryon states with variations of the Borel parameters, the A, B, C, D, E, F, G, H, I, J, K and L correspond to the $\Omega_{ccc}(\frac{3}{2}^+)$, $\Omega_{ccb}(\frac{1}{2}^+)$, $\Omega_{ccb}(\frac{3}{2}^+)$, $\Omega_{bbc}(\frac{1}{2}^+)$, $\Omega_{bbc}(\frac{3}{2}^+)$, $\Omega_{bbb}(\frac{3}{2}^+)$, $\Omega_{ccc}(\frac{3}{2}^-)$, $\Omega_{ccb}(\frac{1}{2}^-)$, $\Omega_{ccb}(\frac{3}{2}^-)$, $\Omega_{bbc}(\frac{1}{2}^-)$, $\Omega_{bbc}(\frac{3}{2}^-)$ and $\Omega_{bbb}(\frac{3}{2}^-)$, respectively.

	This work	[17]	[18]	[19]	[20]	[21]	[22]	[23]
$\Omega_{ccc}(\frac{3}{2}^+)$	4.99 ± 0.14	4.79	4.925	4.76	4.777	4.803	4.67 ± 0.15	4.965
$\Omega_{ccb}(\frac{1}{2}^+)$	8.23 ± 0.13				7.984	8.018	7.41 ± 0.13	8.245
$\Omega_{ccb}(\frac{3}{2}^+)$	8.23 ± 0.13	8.03	8.200	7.98	8.005	8.025	7.45 ± 0.16	8.265
$\Omega_{bbc}(\frac{1}{2}^+)$	11.50 ± 0.11				11.139	11.280	10.30 ± 0.10	11.535
$\Omega_{bbc}(\frac{3}{2}^+)$	11.49 ± 0.11	11.20	11.480	11.19	11.163	11.287	10.54 ± 0.11	11.554
$\Omega_{bbb}(\frac{3}{2}^+)$	14.83 ± 0.10	14.30	14.760	14.37	14.276	14.569	13.28 ± 0.10	14.834
$\Omega_{ccc}(\frac{3}{2}^-)$	5.11 ± 0.15							5.160
$\Omega_{ccb}(\frac{1}{2}^-)$	8.36 ± 0.13							8.418
$\Omega_{ccb}(\frac{3}{2}^-)$	8.36 ± 0.13							8.420
$\Omega_{bbc}(\frac{1}{2}^-)$	11.62 ± 0.11							11.710
$\Omega_{bbc}(\frac{3}{2}^-)$	11.62 ± 0.11							11.711
$\Omega_{bbb}(\frac{3}{2}^-)$	14.95 ± 0.11							14.976

Table 2: The masses of the triply heavy baryon states compared with other theoretical calculations, the unit is GeV.

QCD sum rules, and make reasonable predictions for their masses. The predictions can be confronted with the experimental data in the future at the LHC or used as basic input parameters in other theoretical studies.

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Appendix

The spectral densities of the triply heavy baryon states at the level of quark-gluon degrees of freedom,

$$\begin{aligned}
\rho^A(p_0) &= \int_{z_i}^{z_f} dz \int_{y_i}^{y_f} dy \rho^A(p_0,y,z)\,, \\
\rho^B(p_0) &= \int_{z_i}^{z_f} dz \int_{y_i}^{y_f} dy \rho^B(p_0,y,z)\,, \qquad (15)
\end{aligned}$$

where

$$
\begin{aligned}
\rho^{A,\frac{1}{2}^+}_{QQQ'}(p_0,y,z) &= \frac{3p_0}{8\pi^4}yz(1-y-z)(p_0^2-\widetilde{m}_{QQ'}^2)(5p_0^2-3\widetilde{m}_{QQ'}^2)+\frac{3m_Q^2p_0}{8\pi^4}z(p_0^2-\widetilde{m}_{QQ'}^2) \\
&-\frac{1}{24\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{y(1-y-z)m_{Q'}^2}{z^2}+\frac{z(1-y-z)m_Q^2}{y^2}+\frac{yzm_Q^2}{(1-y-z)^2}\right] \\
&\left[1+\frac{p_0}{4T}\right]\delta(p_0-\widetilde{m}_{QQ'})-\frac{m_{Q'}^2m_Q^2}{192\pi^2p_0T}\langle\frac{\alpha_sGG}{\pi}\rangle\frac{1}{z^2}\delta(p_0-\widetilde{m}_{QQ'}) \\
&-\frac{m_Q^4}{192\pi^2p_0T}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{z}{y^3}+\frac{z}{(1-y-z)^3}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&+\frac{m_Q^2}{32\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{z}{y^2}+\frac{z}{(1-y-z)^2}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&+\frac{p_0}{32\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\left[y-(1-y-z)\right]\left[3+\frac{p_0}{2}\delta(p_0-\widetilde{m}_{QQ'})\right] \\
&+\frac{m_Q^2}{64\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{1}{1-y-z}-\frac{1}{y}\right]\delta(p_0-\widetilde{m}_{QQ'})\,, \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
\rho^{B,\frac{1}{2}^+}_{QQQ'}(p_0,y,z) &= \frac{3m_{Q'}}{8\pi^4}y(1-y-z)(p_0^2-\widetilde{m}_{QQ'}^2)(2p_0^2-\widetilde{m}_{QQ'}^2)+\frac{3m_{Q'}m_Q^2}{4\pi^4}(p_0^2-\widetilde{m}_{QQ'}^2) \\
&-\frac{m_{Q'}^3}{96\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\frac{y(1-y-z)}{z^3}\left[\frac{1}{p_0}+\frac{1}{2T}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&-\frac{m_{Q'}m_Q^2}{96\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{(1-y-z)}{y^2}+\frac{y}{(1-y-z)^2}\right]\left[\frac{1}{p_0}+\frac{1}{2T}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&-\frac{m_{Q'}m_Q^2}{96\pi^2p_0^2T}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{m_{Q'}^2}{z^3}+\frac{m_Q^2}{(1-y-z)^3}+\frac{m_Q^2}{y^3}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&+\frac{m_{Q'}}{8\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\frac{y(1-y-z)}{z^2}\left[1+\frac{p_0}{4}\delta(p_0-\widetilde{m}_{QQ'})\right] \\
&+\frac{m_{Q'}m_Q^2}{16\pi^2p_0}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{1}{y^2}+\frac{1}{z^2}+\frac{1}{(1-y-z)^2}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&-\frac{m_{Q'}}{16\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\left[1-\frac{y}{z}+\frac{1-y-z}{z}\right]\left[1+\frac{p_0}{4}\delta(p_0-\widetilde{m}_{QQ'})\right] \\
&+\frac{m_{Q'}m_Q^2}{32\pi^2p_0}\langle\frac{\alpha_sGG}{\pi}\rangle\frac{1}{z}\left[\frac{1}{1-y-z}-\frac{1}{y}\right]\delta(p_0-\widetilde{m}_{QQ'})\,, \qquad (17)
\end{aligned}
$$

$$
\begin{aligned}
\rho^{A,\frac{3}{2}^+}_{QQQ'}(p_0,y,z) &= \frac{3p_0}{16\pi^4}yz(1-y-z)(p_0^2-\widetilde{m}^2_{QQ'})(2p_0^2-\widetilde{m}^2_{QQ'})+\frac{3m_Q^2p_0}{16\pi^4}z(p_0^2-\widetilde{m}^2_{QQ'}) \\
&-\frac{1}{192\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{y(1-y-z)m^2_{Q'}}{z^2}+\frac{z(1-y-z)m_Q^2}{y^2}+\frac{yzm_Q^2}{(1-y-z)^2}\right] \\
&\left[1+\frac{p_0}{2T}\right]\delta(p_0-\widetilde{m}_{QQ'})-\frac{m^2_{Q'}m_Q^2}{384\pi^2p_0T}\langle\frac{\alpha_sGG}{\pi}\rangle\frac{1}{z^2}\delta(p_0-\widetilde{m}_{QQ'}) \\
&-\frac{m_Q^4}{384\pi^2p_0T}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{z}{y^3}+\frac{z}{(1-y-z)^3}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&+\frac{m_Q^2}{64\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{z}{y^2}+\frac{z}{(1-y-z)^2}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&-\frac{p_0}{48\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle z\left[1+\frac{p_0}{8}\delta(p_0-\widetilde{m}_{QQ'})\right], \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
\rho^{B,\frac{3}{2}^+}_{QQQ'}(p_0,y,z) &= \frac{3m_{Q'}}{32\pi^4}y(1-y-z)(p_0^2-\widetilde{m}^2_{QQ'})(3p_0^2-\widetilde{m}^2_{QQ'})+\frac{3m_{Q'}m_Q^2}{16\pi^4}(p_0^2-\widetilde{m}^2_{QQ'}) \\
&-\frac{m^3_{Q'}}{384\pi^2T}\langle\frac{\alpha_sGG}{\pi}\rangle\frac{y(1-y-z)}{z^3}\delta(p_0-\widetilde{m}_{QQ'}) \\
&-\frac{m_{Q'}m_Q^2}{384\pi^2p_0^2T}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{m^2_{Q'}}{z^3}+\frac{m_Q^2}{y^3}+\frac{m_Q^2}{(1-y-z)^3}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&+\frac{m_{Q'}}{32\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\frac{y(1-y-z)}{z^2}\left[1+\frac{p_0}{2}\delta(p_0-\widetilde{m}_{QQ'})\right] \\
&-\frac{m_{Q'}m_Q^2}{384\pi^2T}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{y}{(1-y-z)^2}+\frac{1-y-z}{y^2}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&+\frac{m_{Q'}m_Q^2}{64\pi^2p_0}\langle\frac{\alpha_sGG}{\pi}\rangle\left[\frac{1}{y^2}+\frac{1}{(1-y-z)^2}+\frac{1}{z^2}\right]\delta(p_0-\widetilde{m}_{QQ'}) \\
&+\frac{m_{Q'}}{192\pi^2}\langle\frac{\alpha_sGG}{\pi}\rangle\left[1+\frac{p_0}{2}\delta(p_0-\widetilde{m}_{QQ'})\right], \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
\rho^{A,\frac{1}{2}^-}_{QQQ'}(p_0,y,z) &= \rho^{A,\frac{1}{2}^+}_{QQQ'}(p_0,y,z)\mid_{m_{Q'}\to -m_{Q'}}, \\
\rho^{B,\frac{1}{2}^-}_{QQQ'}(p_0,y,z) &= \rho^{B,\frac{1}{2}^+}_{QQQ'}(p_0,y,z)\mid_{m_{Q'}\to -m_{Q'}}, \\
\rho^{A,\frac{3}{2}^-}_{QQQ'}(p_0,y,z) &= \rho^{A,\frac{3}{2}^+}_{QQQ'}(p_0,y,z)\mid_{m_{Q'}\to -m_{Q'}}, \\
\rho^{B,\frac{3}{2}^-}_{QQQ'}(p_0,y,z) &= \rho^{B,\frac{3}{2}^+}_{QQQ'}(p_0,y,z)\mid_{m_{Q'}\to -m_{Q'}}, \qquad (20)
\end{aligned}
$$

where $z_f=\frac{p_0^2+m^2_{Q'}-4m_Q^2+\sqrt{(p_0^2+m^2_{Q'}-4m_Q^2)^2-4p_0^2m^2_{Q'}}}{2p_0^2}$, $z_i=\frac{p_0^2+m^2_{Q'}-4m_Q^2-\sqrt{(p_0^2+m^2_{Q'}-4m_Q^2)^2-4p_0^2m^2_{Q'}}}{2p_0^2}$, $y_f=\frac{1-z+\sqrt{(1-z)^2-\frac{4z(1-z)m_Q^2}{zp_0^2-m^2_{Q'}}}}{2}$, and $y_i=\frac{1-z-\sqrt{(1-z)^2-\frac{4z(1-z)m_Q^2}{zp_0^2-m^2_{Q'}}}}{2}$. We can take the limit

$m_{Q'} = m_Q$, and obtain the corresponding QCD spectral densities of the triply heavy baryon states QQQ.

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